**Interesting Problems For Physics / Astronomy 101 & 102**

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INTRODUCTION

The following exercises are suitable for –

* an honors introductory course for non-science majors
* an introductory course for science majors
* a second course for non-science majors

All of them are quantitative, and are at a mathematical level involving nothing beyond exponential arithmetic. Many are quite extensive. None are multiple-choice.

These exercises have evolved out of my years of teaching introductory courses in Astronomy for non-science majors at Amherst College. Surveying the available problems in textbooks, web sites etc I was struck by the need for exercises that students would find interesting, that go beyond the usual and seek to involve the student in more challenging issues.

We have a tendency to think that independent thinking cannot be done by non-science students, and that only advanced science majors have learned enough of the material to begin to think creatively about it. I believe this attitude is false. Here I present exercises designed to move students beyond their normal comfort level, and that give them the opportunity to think about issues somewhat more subtle than they may be used to.

WARM-UP EXERCISES: ESTIMATIONS

Let’s get some practice doing rough estimations, in which it is less important to do a rigorously correct calculation than to get a rough “feel” for the answer.

* If you started right now, could you walk across America before the end of the semester?
* Would it be possible for you to drive to the Sun, and get there before you died of old age? (Assume that there are roads in space, gas stations along the way, etc.)
* The New Horizons spacecraft, launched in 2006, flew past Pluto in 2015. Thereafter it continued on into interstellar space. Suppose New Horizons were traveling straight towards Alpha Centauri, the closest star (actually it isn’t). This star has a parallax of ¾ of a second of arc. Would New Horizons get there before you die?
* Air pressure is 14.7 pounds per square inch. The classic movie “2001: A Space Odyssey” includes a scene in which an astronaut leaves a space capsule and heads back into his main spacecraft – without a space suit. So for a few brief moments, he is holding his breath in the vacuum of outer space. I have always wondered if this portion of the movie were scientifically inaccurate. I wonder if actually the air pressure in the astronaut’s lungs might be enough to explode him. Am I correct?
* Which is greater: the amount of water in the Atlantic Ocean, or all the tears that have ever been shed since homo sapiens appeared?
* If everyone started walking eastward at the same moment, would the length of the day change noticeably?

*Note: In doing these estimations it is essential that you discuss in detail your methods of calculation, the assumptions you made, the reasons why you made these assumptions, and so forth. An answer, even a plausible answer, not accompanied by such a careful discussion will get no credit.*

NAKED EYE ASTRONOMY

Summer and Winter Constellations

As you know, constellations are visible at only certain times of year. Cygnus is visible in summer, Orion in winter. The object of this problem is to understand why.

Here is a diagram showing the Earth, the Sun, and Earth’s orbit about the Sun:

Earth

Sun

You are here

Up for you

Your horizon

(In this diagram, the dotted circle is the Earth’s orbit, the solid circle is the Earth, and the little dot indicates your location on the Earth, as you gaze upwards at the sky.)

* Notice that, in this diagram, your “up” direction points directly away from the Sun. What time is it at your location?
* Suppose Orion is directly overhead for you at this moment. Indicate on the above diagram the direction to Orion.
* Now draw a similar diagram showing the configuration 6 months later.
* Explain why your new diagram demonstrates that Orion will no longer be visible to you.
* Return to these two diagrams and indicate upon them the direction to Cygnus. Explain why Cygnus cannot be seen now, but will be visible 6 months later.

Mars as observed from the Earth (2 problems)

(I)

Mars is the second-closest planet to us. Let us get a feel for how easy – or how difficult – it is to observe the planet. We will suppose that we are using a telescope situated on the Earth.

Here is a diagram illustrating the Earth (the inner planet) , Mars (the outer planet) and their orbits:

1 AU

1.5 AU

As you can see, depending on where the two planets lie in their orbits, our distance to Mars varies by quite a bit.

* What is the minimum distance between us and Mars? What is the maximum?
* Find the angular diameter of Mars as seen from the Earth in these two configurations.
* To get an intuitive feel for your answers, measure the diameter of a dime and then calculate how far you must be from it in order for it to subtend these angles. Express your answers in yards.
* Suppose you are in a configuration in which Mars is directly overhead at midnight (this is called Opposition). Draw a diagram showing that, in this configuration, Mars is closest to the Earth.
* Suppose you are in a configuration in which Mars is at its greatest distance from us. At what time of day is Mars directly overhead? Is Mars visible from Earth when it is in this configuration?

(II)

Let us now think about a configuration in which Mars is directly overhead at some other time – say, at 9 PM. Our goal is to figure out where Mars would be in its orbit in such a situation.

Here is a diagram. In it, we are looking down upon the Earth from a location directly above the north pole. From such a vantage point, the Earth rotates counter-clockwise, and the rotation axis points directly out of the screen. We imagine three observers located along the equator:

Earth as seen from directly above the North pole:

Location B



Us

Sunlight

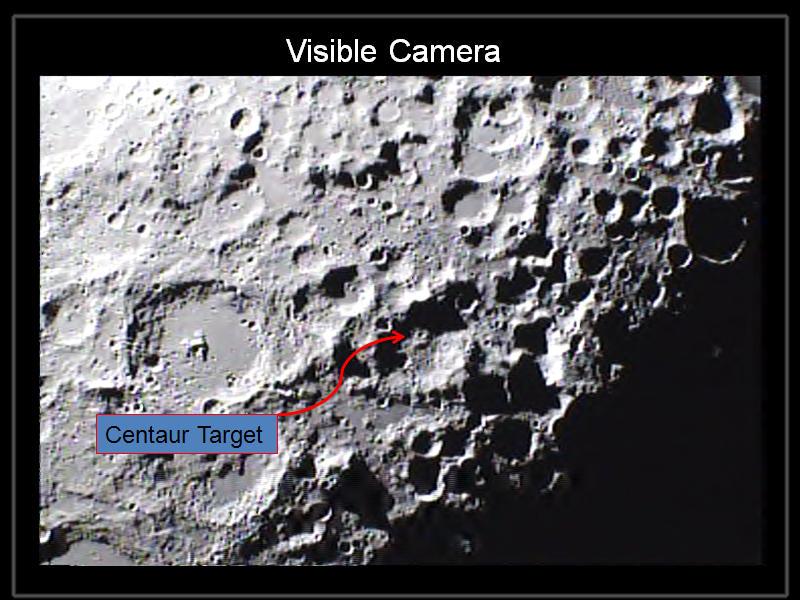
Location A

* What time is it at locations “A” and “B”?
* Where is the Sun as seen from locations “A” and “B” – high in the sky, on the horizon, etc?
* Suppose that for us Mars is directly overhead at 9PM. Explain why the angle  in this picture is 45o
* Draw a diagram illustrating the Sun, the orbits of the Earth and Mars, and the two planets’ locations in such a configuration.
* On your diagram, what is the angle between the Sun and Mars ?

Viewing Configurations for the LCROSS Impact

In 2009 NASA slammed its LCROSS mission (the Lunar CRater Observation and Sensing Satellite) onto the Moon. The goal was to “shoot a bullet” at our satellite, observe the debris from the impact, and thus search for water on the Moon. The mission succeeded and water was indeed detected.

Cabeus, a crater close to the Moon’s south pole, was selected as the site of impact. Here is an image of the impact site:

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Although it is not indicated on this image, the direction to the Earth at the moment that LCROSS hit was roughly towards the 11 o’clock position (i.e. towards the upper-left-hand vertex of the image). As seen from Cabeus, the Earth lay close to the Moon’s horizon in this configuration.

* Take a close look at the shadows on the image, and explain why they demonstrate that, at the moment of impact, the direction to the Sun was just about the same as the direction to the Earth.
* What was the phase of the Earth that you would have seen had you been standing in Cabeus? Would it have been a full Earth, crescent Earth, or what?
* At the moment of impact, many telescopes on Earth were trained on the Moon, hoping to observe the debris. What would the Moon’s phase have been, as seen by these astronomers?

The impact took place at 11:31 AM London time.

* Would astronomers in England have been able to observe the impact?
* Suppose you had been an astronomer who badly wanted to observe the impact. Where on the Earth should you have journeyed to in order to do so?
* When you observed the impact, would it have been daytime or nighttime from your location?

The Sky as Seen From the Moon

What would the sky look like if you lived on the Moon?

The Moon orbits about the Earth, taking one month to do so, and the Earth orbits about the Sun, taking a year to do so. Furthermore, the Moon spins about on its axis, taking just a month to complete one rotation, so that the Moon keeps its same side always facing the Earth. Suppose we imagine two astronauts. The first, named Chris, lives on a place on the Moon which faces the Earth. The second, named Pat, lives on a place on the Moon which faces away from the Earth:

Moon’s orbit about the Earth

Chris now Chris half a month later

Earth

Pat now Pat half a month later

* Discuss the appearance of the sky as seen by Chris and Pat. Your discussion should be very well written, and very extensive. You should include lots of figures, clearly labeled. Discuss the views these astronauts have of the Earth, the Sun, and the various constellations. Discuss how these views change as a month passes, and as a year passes. There is no single list of things which I would like you to think about: be creative, and think of lots of interesting things. You will get extra credit for inventiveness!

NEWTON’S LAWS. GRAVITY AND ORBITS

Weight

Let’s get some practice using Newton’s formula for the force of gravitation.

* Use it to calculate the weight of a 70-kilogram astronaut on Neptune. Give your answer in pounds to get an intuitive feel for it.
* In designing a mission to an asteroid, it is important to know how much a space probe would weigh on the asteroid. Unfortunately we do not know most asteroids’ masses. But we do know how big they are, and that they are made of solid rock -- and rocks typically have densities of about 2 grams / cm3. Use this to estimate the weight of a 200-kilogram space probe on an asteroid 20 kilometers across. Give your answer in ounces to get an intuitive feel for it.
* Based on your answer, speculate on how a robotic spacecraft might move from place to place across the surface of an asteroid.

Gravity and the Orbit of the Moon

Calculate the gravitational attraction

* Between the Earth and the Moon
* Between the Sun and the Moon

Based on your answer, discuss the orbit of the Moon

[Because the Moon’s distance from the Earth is so much less than an A.U., you can assume the distance between the Sun and the Moon is the same as the distance from the Sun and the Earth.]

Centripetal Acceleration: Thrill Ride

Recall the formula for centripetal acceleration:

acent = v2 / R

where v is the object’s velocity and R the radius of the circle it is moving in. We also know from Newton’s second law that, in order to be accelerated like this, there must be a force

F = m acent

acting on the object, where m is the object’s mass.

Suppose the object is you, and that your mass is 70 kilograms. Suppose you are on one of those terrifying rides in an amusement park, in which you are whirled around in a circle -- you sit strapped in a chair, which is at the end of a long arm which whirls about. Suppose the thing whirls you in a circle once every 4 seconds, and that the length of its arm is 4 meters.

• What is your velocity?

• What force does the arm exert on you? Calculate your answer in dynes but then convert it to pounds.

Centripetal Force on a Car Making a Turn

Suppose you are driving a one-ton car.

* Remember: this means that the force of gravity from the Earth on the car is a ton. What is the car’s mass in grams?

Now suppose you make a right-hand turn. As you make that turn, you are temporarily moving in a circle – you make a quarter of a turn around the circle, and then straighten out:

R

Let us say that the radius of this circle is about ten feet, and you are driving at ten miles per hour.

* What is the centripetal force on the car? Calculate it in dynes, but then convert it to tons.

[I must say that, when I did this calculation, the answer surprised me. I guess this is why tires squeal if you turn too rapidly!]

F = MA:

A Spacecraft to Mars

A recent mission to Mars had an on-board thruster capable of exerting a force of 88 pounds, which in space decelerated the spacecraft by nearly 3,000 kilometers/hour after firing for 30 minutes.

* Find the spacecraft’s acceleration. Give your answer in cm/sec2
* Find the spacecraft’s mass. Calculate your answer in grams.
* What object in daily experience has such a mass?

Geosynchronous Satellites

If you use a satellite dish for TV reception, that dish is pointing at a geosynchronous communications satellite. These satellites appear to hover motionlessly above the Earth. Of course they do not: if they really were motionless, they would just fall back down. In reality, they are orbiting about the Earth once every 24 hours.

* Such satellites are always placed in orbits lying in the equatorial plane of the Earth. Why is this? Draw a diagram showing why such a satellite, even though it orbited once a day, would appear to hover motionlessly as seen from the ground.
* Suppose alternatively that the satellite were placed in an orbit passing over the North and South poles. Draw a diagram showing why such a satellite, even though it orbited once a day, would most definitely not appear to hover motionlessly as seen from the ground.
* Recall the formula for the velocity of a satellite in a circular orbit of radius R about an object of mass M: Vorbit = √GM/R. Use it to find a formula for the length of time it takes a satellite to orbit once.
* Show that your formula can be written



where K is a constant. [Note that you have now derived Kepler’s third law for the special case of a circular orbit. Congratulations!]

[*Note* Don’t worry about the “K” here. The above formula is appropriate for cgs units. Kepler expressed his law in AU and years. It turns out that, if you change variables in your answer to AU and years you will find that in that system of units K=1.]

* Now find a formula for the distance such a satellite must lie from the Earth’s center.
* Your formulae are also valid for a spacecraft orbiting any other body. Let’s design an orbit of a spacecraft circling about Mars which will put it perpetually over one of the Mars Rovers – a “Mars-o-synchronous orbit.” How far must such satellite lie from Mars’ center? To get an intuitive feel for your answer, express it in miles.

The Sun rotates once every 24 days.

* Suppose we wished to launch a satellite designed to study one and only one region of the Sun. Such a satellite would have to be in a “Sun-o-centric orbit.” How far from the Sun would such an orbit lie?
* Might such a satellite be in danger of being fried by the intense heat of the Sun?

Shooting Down a Spy Satellite: The Mass of the Earth

How do we “put the whole Earth on a bathroom scales” and measure its mass? By using the Circular Orbit Formula in reverse! Remember the formula:



which gives the velocity of an object orbiting in a circle of radius R about a body of mass M. We can solve this to give the mass M if we know the orbiting body’s velocity and orbital radius.

A while ago I read a news report of how the Defense Department shot down one of its own spy satellites that was spiraling into the Earth. Most of the project was classified, but these reports gave two interesting facts: the satellite orbited the Earth 16 times a day, and it was 130 miles up.

* From this data, find the mass of the Earth

LIGHT: INTRODUCTORY

Hypothesis testing:

Does Light Travel in Straight lines?

Hypothesis testing is an essential element of science. Here we get some practice in this essential procedure.

Science always proceeds in the following manner:

1. One performs observations, and (hopefully!) notices an interesting phenomenon of some sort
2. One then proposes a theory to account for the phenomenon
3. One then tests the theory. This is done by forcing the theory to make a prediction, and then testing to see if the predicted behavior is observed.

In what follows we will follow this procedure, to see how it works.

(1) The observations

Here is a class exercise: go outside on a sunny day and measure the length of your shadow.

It is important that a lot of students do this, and that they all do it at the same time. Then each student computes R, which is defined to be the ratio between the length of the student’s shadow and the length of the student (i.e. his/her height).

Here now is the interesting phenomenon: every student gets the same value for R.

Clearly, we will need to invent a theory to account for this phenomenon. But before we do so, a critical issue needs to be addressed: if the observations were honest, the actual values of R will not be exactly equal. They will just be nearly equal. Does this arise because of measurement error, or is our purported discovery merely a mistake? In order to answer this question we need to know how big are the errors of each measurement.

* A topic for class discussion: how can you measure the error of the measurement of R?
* Carry out your test

In what follows, we will assume that this has been done, and the conclusion has been reached that our purported phenomenon really exists (within the limits of your measurements.)

(2) The Theory

Here is a theory to account for the phenomenon: light travels in straight lines.

* Using the following diagram, explain how the theory accounts for the phenomenon:



Shadow, length L

Sunlight

Student, height H

(3) Testing the theory

It is never enough to propose a theory. One must always test the theory. As we have seen, the way to do this is to ask the theory to make a prediction.

Here is a prediction of our theory: According to the theory, if the various measurements of R are carried out at different times of day, the various R s will not be equal.

* Explain why the theory makes this prediction.
* Design a series of observations that will test the prediction
* Carry out the observations (with careful attention to measurement errors)

Finally here is a further test of the theory: it makes a definite prediction for how R depends on the time of day.

* What is this prediction?
* Design a series of observations that will test the prediction
* Carry out the observations (with careful attention to measurement errors)

More on Hypothesis testing:

Does Light Really Travel in Straight lines?

In the previous problem you measured the length of your shadow and computed R, the ratio between the length of your shadow and the length of you. You found that R was the same for everybody. And you developed a theory to account for this discovery.

Part of that theory is the assumption that light travels in straight lines. But is this assumption really true? The object of this problem is to design a series of measurements which can experimentally test that assumption.

To make things definite we will propose a new theory of light, and see what it predicts about R. Then we will study the predictions of this new theory, to see if your measurements show the theory to be true or false.

Here’s the new theory: light travels in a straight line so long as it is 1 centimeter or higher above the ground. But once a ray of light reaches a height of 1 centimeter, it falls straight down.

(I want to make clear that this is not a theory to be taken seriously. We are only examining it in order to see how science tests a hypothesis.)

* Explain why this theory predicts that objects shorter than 1 centimeter cast no shadows.

Now let’s move on to objects taller than 1 centimeter. Let’s call this critical height of 1 centimeter L. Here’s a picture of the shadow (length L) cast by an object whose height (H) is more than L.

sunlight



Object, height H

L

Shadow, length L

* Prove that, according to this new theory, the length of a shadow is given by L = (H – L) / tan ()
* Prove that our ratio R is therefore given by R = [1 – L/H] / tan ()

Notice that according to this formula R depends on H: different values of H yield different values of R. Let’s explore this:

* Suppose is 30 degrees. Calculate our new theory’s predicted values of R for objects whose height H is equal to
  + 0.5 centimeter
  + 1.5 centimeters
  + 2 centimeters
  + 10 centimeters
  + 100 centimeters
  + 1000 centimeters

You will find an interesting result: our new theory predicts that R should be nearly constant for tall objects, but that R is very definitely not constant for short objects. So that’s how to test the theory: measure R for short objects.

* So did the class measurements test this theory?

LIGHT: THE INVERSE SQUARE LAW

How faint a light can we detect?

Any detector of light is only capable of detecting that light if it has a certain minimum apparent brightness. If a light is fainter than this limit – that is, if its flux is below a certain critical limit – we simply don’t detect it. Let us call this minimum flux fminimum.

Some time ago I went outside on a dark night and performed an experiment: I placed a 100 Watt light bulb along a deserted roadside and drove farther and farther away from it. Eventually I got to about 8 miles from the light bulb, and I found that I could no longer see it. (I emphasize that it was a very dark night, and that I had carefully dark-adapted my eyes. You can’t do such a thing in normal circumstances.)

* What is the minimum detectable flux fminimum for my eyes?
* A major telescope is capable of detecting a minimum flux fminimum of about 10-14 ergs/cm2 sec. From how far away could such a telescope detect that light bulb? Give your answer in miles to make it intuitively comprehensible.
* If there were a 100 Watt light bulb on the moon, could such a telescope spot it?

Using the Inverse Square Law

Let’s get a little practice using the inverse square law to think about how dim sunlight is on Neptune, the most distant planet in the solar system.

* Find the apparent brightness of the Sun as seen from Neptune – that is, the flux of sunlight on Neptune.
* To get an intuitive feel for your answer, find how much dimmer sunlight is on Neptune than on the Earth – that is, find the ratio between the flux of sunlight on Neptune and that on the Earth.
* To continue getting an intuitive feel for your answers: calculate how far you have to be from a 100-watt light bulb in order for the flux you receive from it to equal (a) that we on Earth receive from the Sun, and (b) that Neptune receives from the Sun. Give your answer in feet.
* Unmanned spacecraft use very small quantities of power. Supposed one required only a kilowatt, and suppose we wanted it to get this power from solar energy once it was out near Neptune. How big must its solar panel be in order to capture this much energy? Begin by calculating its area, but than find the length of one side of the solar panel (assume it is square). Express your answer in feet to get an intuitive sense of how big it is.

[Your answer to this last questions tells you why missions to the outer solar system don’t use solar power: the required size is just too large. Instead they use nuclear energy.]

Sunbathers at the Equator and the Poles

Why do we get sunburns at the equator, but not at the poles?

You might speculate that the reason is that the equator is closer to the Sun, making the flux of sunlight at the equator greater than at any other location. In fact this speculation turns out to be wrong: while the flux is indeed greater, it is only a little bit greater – not enough to make a difference. The object of this problem is to demonstrate this.

Let’s consider two sunbathers: one at the north pole, and one at the equator. Let’s call Dpole the distance between the Earth’s north pole and the Sun, and Dequator the distance between the equator and the Sun. Clearly Dequator is less than Dpole, which makes sunlight indeed brighter at the equator than the pole. But how much brighter?

* Referring to the following figure, explain why the difference between Dpole and Dequator is almost exactly R, the radius of the Earth.

One sunbather

To the sun

R

The other sunbather

equator

* The Astronomical Unit is defined to be the distance between the Earth’s center and the Sun. Explain why Dpole is almost exactly one Astronomical Unit.
* Calculate the flux of sunlight for each sunbather, in ergs / cm2 sec.
* Find the ratio of these two fluxes: the flux at Dequator divided by the flux at Dpole

You will find that this ratio is only a tiny bit greater than one. This proves that the flux of sunlight at the equator is only a little more than that at the pole: not enough to give people sunburns.

* Why, then, do we get sunburns at the equator but not the poles? See if you can think of an answer.

Solar Power

Let us study the potential of solar power to supply the energy needs of our modern industrial civilization.

The flux of sunlight we receive here at the earth

ffrom sun = 1.36 X 10 6 ergs / cm2 sec

is the amount of energy falling on each square centimeter each second. If we want to find the total amount of solar energy falling on the entire earth, you might think that we should multiply ffrom sun by the number of square centimeters on the earth – i.e. the earth’s surface area. But this would be wrong.

Here’s a picture that explains why.

sunlight

As you can see, half of the earth does not receive any sunlight at all, so we should not count the area of that half. Furthermore, even on the sunlit half, many of the square centimeters are oriented at slant angles to the incoming sunlight. All this makes it hard to decide what we should multiply ffrom sun by.

Here’s a way to figure this out: think about the shadow cast by the Earth.

* Explain using diagrams why the shadow cast by a sphere has area r2 (where r is the radius of the sphere) and why this area tells us the amount of light intercepted by the sphere.
* Find then the total number of ergs falling on the earth each second.

Very roughly the human race uses about 10 20 ergs of energy each second.

* How many ergs has the human race used in the last 10 years?
* Suppose we covered the entire earth with solar panels, and that they captured the sunlight with perfect efficiency – i.e. that they turned each erg of solar power into an erg of electricity. For how much time would we have to operate them in order to collect this much energy? Convert your answer to hours to get an intuitive feel for it.

LIGHT: WAVES AND THE DOPPLER EFFECT

Qualitative Doppler Effect # 1

A car drives in the square pattern illustrated below. You are far off to the right.

1

4

2

To you

3

* On board the car is a guitarist continually strumming the note Middle C. At each of the four stages 1, 2, 3 and 4 are the notes you hear coming from the guitar higher than middle C, lower than middle C, or the same as middle C?
* Are the wavelengths of the note you hear (at each of the four stages 1, 2, 3 and 4) longer, shorter or the same as that of middle C?
* The car has a color intermediate between red and blue. Are the car’s colors you see more nearly red, more nearly blue, or unchanged at each of the four stages?
* And finally, how do the wavelengths of the light you see change as the car moves along its path?

Now suppose that the car keeps moving in this pattern, but that you start moving to the left at constant velocity in a straight line As a matter of fact, imagine that you are moving at exactly the same speed as the car.

* Answer the 4 above questions for this new situation. In your answers, make sure you compare your answers to those in the 1st case.

And finally, suppose that you move at the same speed, but in a path synchronized with that of the car: when it moves to the right, so do you, and when it moves down so do you. So now both of you are moving in that square pattern.

* Answer the 4 above questions for this 3rd situation. And again, in your answers, make sure you compare your answers to those in the 1st case.

Qualitative Doppler Effect # 2

In the following diagrams, the box indicates an observer studying the light from the star. In each of the following situations, indicate whether the observer sees a red shift, a blue shift or no shift in the wavelength of the star’s light. (Make sure you explain your answers!)

A

B

C

D

E

Radio Waves and the Doppler Effect

The college where I teach has a student-run radio station whose radio waves have a frequency of 89.3 million cycles per second.

* What are their wavelengths? Give your answer in feet to get an intuitive feel for it.
* Suppose you have a radio in your car tuned to a different station, which broadcasts at 91.1 million cycles per second. How rapidly do you have to move in order to pick up the college’s 89.3 station? Convert your answer to miles per hour to get an intuitive feel for it.
* In what direction should you drive?

NUCLEAR PHYSICS

Radioactive Decay and Age Dating

Consider the decay of the isotope of Uranium, U235:

U235 🡪 Pb207 + 7 He4

The half-life for this decay is τ = 0.7 billion years.

Suppose we consider a rock that initially contains N nuclei of U235. As time passes, more and more of these nuclei decay, and more and more Lead and Helium nuclei appear. Let us follow the progress of these decays, to see how they allow us to build a “clock.”

* To this end, fill in the entries in the following table

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| time | # of Uranium nuclei in the sample at the beginning of this period of time | # of Lead nuclei in the sample at the beginning of this period of time | # of Helium nuclei in the sample at the beginning of this period of time | #He / #U in the sample at the beginning of this period of time | # of decays that will take place in the next τ years |
| 0 | N | 0 | 0 | 0 |  |
| τ |  |  |  |  |  |
| 2 τ |  |  |  |  |  |
| 3 τ |  |  |  |  |  |

(Be warned: you must explain in detail the reasoning you went through in arriving at your answers. You will get no credit for simply filling in this table without this explanation – even if your answers are correct!)

* Plot your answers
* Suppose a rock is found for which the ratio #He / #U is measured to be 30. What is the age (in years) of this rock?

Radioactive Decay of a Mythical Element

The decay of Uranium is often used as a means of finding the ages of rocks. Here we get some more practice with this method by considering the decay of a mythical new element that we’ll call (why not?) Galaxium.

Galaxium, with a mass of 258 atomic mass units, is found to decay into two atoms of Indium (mass 115) and seven of Helium (mass 4) with a half-life of 2.1 billion years:

Ga258 🡪 2 In115 + 7 He4 t = 2.1 billion years

Suppose we begin at time =0 with N atoms of Galaxium, and none of Indium or Helium.

* Calculate how many atoms of Galaxium, Indium and Helium there will be after 1,2 and 3 half lives. Find also the ratio of the number of He to Ga atoms at these times. (Make sure you explain clearly your reasoning: a set of answers with no such explanation will get you no credit.)
* Make a plot of the ratio He / Ga as a function of time.
* Suppose a sample is found for which the ratio of Helium to Galaxium is 30. What is the age in years of this sample?

Be Careful With That Cup Of Coffee

In this problem we want to calculate how much energy would be released if all the Hydrogen in a cup of coffee were to undergo nuclear reactions to form Helium. The answer is going to be frightening.

Recall that, every time 4 Hydrogen nuclei fuse to form one Helium nucleus, Ereleased = 4.23 X 10 – 5 ergs of energy are released. So all we need to do is figure out how many Hydrogen nuclei a cup of coffee contains. To do this, let us assume that coffee is pure water – that’s not exactly true, but it’s almost true.

* In the H2O molecule, the H has the mass of a proton, and the O has the mass of 16 protons. Find the mass in grams of one H2O molecule.
* Now we need to find the mass of a cup of water. Since the density of water is one gram per cubic centimeter, this means we need to find out the number of cubic centimeters in a cup. Look this up, or measure it, or whatever.
* How many H2O molecules does a cup of water contain?
* How many Hydrogen atoms does it contain?
* How many Helium nuclei can be formed from all these Hydrogen atoms?
* How much energy would be released were this to happen? Calculate your answer in ergs, but then convert it to kilotons.
* Just for fun, let’s also calculate how much energy would be released if all the Hydrogen in your body were to fuse to form Helium. Are you a Hydrogen bomb?

Chemical and Nuclear Reactions:

Does a Fire Lose Mass?

As you know, energy emission from stars is powered by nuclear fusion reactions in their cores. These reactions involve Einstein’s famous E = MC2: when two nuclei react to form a product, the product’s mass is less than the sum of their masses. The lost mass is converted to energy, which is the energy released in the reaction.

But Einstein’s formula is not just confined to nuclear reactions. In reality, any reaction which involves a release of energy involves an accompanying loss of mass. Let us apply this principle to the simple case of a fire in a fireplace. According to Einstein, the energy E released by the burning of wood is accompanied by a mass loss M = E/C2.

You may think that this makes plenty of sense. After all, an armload of wood weighs a good deal; but once that wood has been burned in a fireplace, all that is left is a pile of ashes -- and the ashes weigh a lot less than the wood from which they came. So it seems that we are directly witnessing here Einstein’s principle in action.

But in fact this is wrong. While it is certainly true that a fire involves the transformation of mass into energy, the quantity of mass which is lost is miniscule. The object of this problem is to demonstrate this.

Experiments reveal that when one gram of wood is burned 1.5 X 1011 ergs are released. Suppose you burn 20 pounds of wood in a fireplace.

* How many ergs of energy does this release?
* According to Einstein’s formula, this means an amount of mass M has vanished. Calculate M in grams.
* Is your result for M large enough to account for the difference between the weight of the wood you dumped into the fireplace and the weight of the ashes left after the wood was burned up?
* If not, how do you account for the fact that ashes are so light? Where did all the mass go?
* In the nuclear reactions which power the shining of a star like the Sun, 0.7% of the mass of the Hydrogen fuel turns into energy. Do the same calculation for the case of burning firewood: what percent of the mass of the wood turns into energy?

STARS

Apparent Brightness, Parallax

and the Distance to Sirius

Sirius has the largest apparent brightness of any star in the sky.

Before people had discovered the phenomenon of parallax they had no idea how far away Sirius is. But suppose they had guessed that Sirius is just like our Sun. They would then have explored the consequences of assuming that the luminosity of Sirius equals that of our Sun.

* The measured flux of light from Sirius is 1.2 X 10 – 4 ergs/ cm2 sec. From this, calculate how far away Sirius would be based on our assumption. Give your answer in cm, but then convert it to light years.
* Predict what the parallax of Sirius would be in this case.
* In fact the measured parallax of Sirius turns out to be 0.38 seconds of arc. What is the real distance to Sirius? Again, give it both in cm and light years.
* What is the real luminosity of Sirius? Give your answer in ergs/sec. What is the ratio between the luminosity of Sirius and that of the Sun?

Energy From the Sun

Our world-wide industrial civilization uses about as much power as 150 billion 100-Watt light bulbs.

* How long would it take for the Sun to emit enough energy to fuel our world-wide energy needs for a century?

X-Ray Emission From the Sun

Our own Sun is observed to emit X-rays. The measured flux here on the Earth of X-ray energy from it is about 0.8 ergs / cm2 sec

* Find the luminosity of the Sun in X-ray energy – i.e. the number of ergs of X-ray energy it emits per second.
* Does the Sun emit more energy in the X-ray or the visible wavelengths? What is the ratio between these two forms of energy emission?

Nuclear-Burning Lifetimes:

What Kind of Star Forms A Black Hole?

When a star runs out of nuclear fuel it collapses, possibly to form a black hole. The object of this problem is to calculate how long it takes for a star to run out of fuel. You will end up showing that any black holes that exist today could only have been formed from high mass stars.

Recall that

* a star continues shining until it has exhausted all its nuclear fuel
* each gram of nuclear fuel releases 6.3 X 1018 ergs of energy
* a star like the Sun is only capable of using 10% of its mass as nuclear fuel (the remaining 90% never gets hot enough to undergo thermonuclear reactions). To make things simple, let’s assume this is true for stars of other masses.

We will work with a variety of stars. Here is the data you will need:

|  |  |
| --- | --- |
| Mass  (Solar masses) | Luminosity  (solar luminosities) |
| 0.1 | 3.16 X 10 - 4 |
| 0.3 | 1.48 X 10 - 2 |
| 1 | 1 |
| 3 | 4.68 X 10 1 |
| 10 | 3.16 X 10 3 |

As you can see, the higher-mass stars burn up their nuclear fuel a lot more vigorously than the lower-mass ones: this means they will run out of fuel a lot sooner.

* Your task is to calculate the nuclear-burning lifetimes of these stars. Give your answer both in seconds and years.
* Use your results to create a graph of lifetime versus mass.
* Given that our Milky Way galaxy is about 10 billion years old, which mass stars have had time enough to form black holes by now?

Mass Defects: Nuclear Reactions in Red Giant Stars

Red giant stars shine, not by converting Hydrogen to Helium, but by converting Helium to Carbon. The reaction is

3 He4 🡪 C12

The mass of the C12 nucleus is 1.9922 X 10 -23 grams and the mass of the He4 nucleus is 6.645 X 10 -24 grams.

* How much mass vanishes each time three Helium nuclei combine to form one Carbon nucleus?
* How much energy is released each time this happens?
* What percent of the mass of those three Helium nuclei vanishes each time they combine to form one Carbon nucleus?

Where Does the Helium in

the Sun Come From?

Measurements reveal that the Sun contains a good deal of Helium: roughly ¼ of the Sun’s mass is this element. We know that the Sun shines by converting Hydrogen into Helium, so you might think that the Helium in the Sun comes from those nuclear reactions. Amazingly, this is not so. Your task in this problem is to show that the Helium in the Sun cannot have been made by these nuclear reactions.

* Find the total mass (in grams) of all the Helium in the Sun
* Find the number of Helium nuclei “Nactual” in the Sun.

Every time the reaction 4 H 🡪 He happens, E = 4.23 X 10 -5 ergs of energy are released.

* From the measured luminosity of the Sun, calculate how many Helium nuclei are being formed within the Sun each second.
* How many Helium nuclei “Nformed” has the Sun formed in the 4.5 billion years of its history?
* Find the ratio between Nformedand Nactual.

Your result is going to be that Nactual is a lot bigger than Nformed: the total amount of Helium that the Sun has created in all its billions of years of shining is only a small fraction of the amount of Helium it contains.

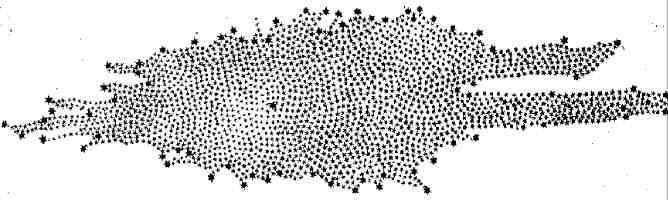
So the question is: where did all that Helium come from? We believe that it was created by nuclear reactions during the Big Bang.

GALACTIC AND EXTRAGALACTIC ASTRONOMY

A Danger With Herschel’s Method

of Finding Our Location in the Universe

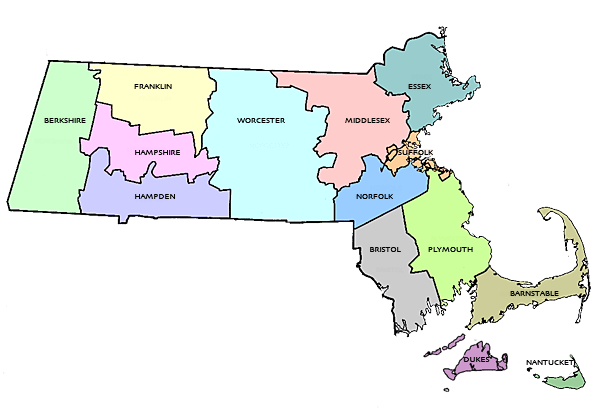
As you know, Herschel attempted to find our location in the universe by counting the number of stars in various directions. He found lots of stars in some directions, and fewer in others. This led him to his famous “grindstone model” of the cosmos:



And as you know, this model was entirely incorrect.

Two factors had led Herschel astray. One was the existence of dark obscuring clouds. Another, which we will study here, had to do with his assumption that stars are spread uniformly about us. This problem illustrates the dangers of that assumption.

Here is a map of Massachusetts, showing the various counties:



The populations of each county, from a recent U.S. Census, are:

County County Population

Barnstable County 222230

Berkshire County 134953

Bristol County 534678

Dukes County 14987

Essex County 723419

Franklin County 71535

Hampden County 456228

Hampshire County 152251

Middlesex County 1465396

Nantucket County 9520

Norfolk County 650308

Plymouth County 472822

Suffolk County 689807

Worcester County 750963

Suppose you live in Worcester County, Massachusetts and you wish to know your location. You decide to do this by a variation of Herschel’s method: you count the number of people living in various directions. Unfortunately this method is going to lead to a false result, because the eastern parts of Massachusetts are far more densely populated than the western parts.

* By consulting the above map, find the total number of people who live in various directions from Worcester county. By using your results, where would you conclude you lived were you to use Herschel’s method -- i.e. if you assumed that people were spread uniformly about Massachusetts?
* Does Herschel’s method yield a correct result?
* What could Herschel have done to find out if his assumption was valid?

Calibrating the Period / Luminosity Relation

For Cepheid Variable Stars

Recall that Cepheid variable stars were first found in a nearby galaxy, and it was realized that the longer the period of such a star the greater its apparent brightness. People immediately realized that this would provide a new way to measure distances.

Unfortunately, since nobody knew the distance to that galaxy, nobody could find the connection between apparent brightness and luminosity in order to calibrate this method. Historically, this problem was eventually solved by finding Cepheids in a relatively nearby globular cluster of stars, and then measuring the distance to this cluster (by using the parallax technique). The object of this exercise is to follow through the details of this process.

Here are three Cepheid variable stars in our globular cluster:

|  |  |  |
| --- | --- | --- |
|  | Period of variation (days) | Average flux (units are  10 – 5 ergs/sec-cm2) |
| Cepheid #1 | 2 | 1.68 |
| Cepheid #2 | 4 | 2.7 |
| Cepheid #3 | 9 | 5.0 |

The parallax of this cluster was eventually measured to be 0.057 seconds of arc.

* Find the distance to this cluster, first in parsecs and then in cm.
* find the luminosities of the above three Cepheids. Express your answer in Solar Luminosities.
* Draw a graph of the luminosity (in Solar Luminosities) versus period (in days) for Cepheid variable stars. This is your very own Period – Luminosity relation for Cepheids.
* A Cepheid is found in a very distant galaxy whose period is 7 days, but whose flux is a mere 6.5 X 10 -15 ergs/sec-cm2. Find the distance to this galaxy. Express your answer in light years.

Using the Cepheid Period / Luminosity Relation

To Measure Distances

Now let us turn to a different object – a globular cluster of stars.

You may have seen an image of such a cluster. They are very beautiful, and some are close enough to be seen by the naked eye. If you have good eyesight, you might enjoy trying to spot one: any astronomical map will show you where to look. The problem in seeing them is not that they are small – in fact they are not small at all! It is that they are very faint. If you do spot one, what you see will appear to the naked eye to be a faint, pearly glow.

This problem concerns a hypothetical cluster which appears to be just 11.4 times as big across as the Moon. Since the Moon is 1/2 of a degree across, this means that the cluster subtends an angle of 5.7 degrees. The flux of light we receive from it is measured to be

fcluster = 1.43 X 10  - 4 ergs / cm2 sec

(I emphasize that this is the flux from the entire cluster – the total of the flux from all the stars within it.)

So far this is all we know about the cluster – and it isn’t very much! But then a Cepheid variable star is discovered within it. The star takes 4 days to go through its cycle of variation, and the flux we receive from that single Cepheid variable star is found to be

f Cepheid = 4.81 X 10 – 7 ergs / cm2 sec

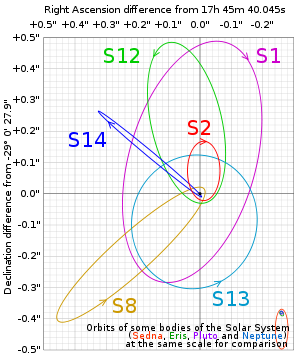
Your task is to calculate

* The distance to the cluster.
* The total number of stars within the cluster, assuming that all of them (except that one Cepheid!) have a luminosity equal to that of the Sun.
* The radius of the cluster.

A Black Hole at the Center of Our Galaxy?

In recent years evidence has been found that our very own galaxy has a giant black hole at its center. The evidence comes from tracking the orbits of stars about the galactic center. The enormous force of gravity from the hole bends the orbits of these stars into very tight ellipses: by applying Kepler’s third law to these orbits, we can measure the mass of the hole.

The orbits are shown in the graph below:



The purported black hole lies at the origin of this graph.

Let us concentrate on the star S2: in what follows we will assume that we are seeing its orbit face on. While this is not in fact true, it simplifies the analysis and gives us at least a rough idea of the true situation.

* From the graph, measure the major and minor axes of this star’s orbit, and its distance of closest approach to the hole. (All these measurements will be in seconds of arc.)
* Given that the galactic center is some 25,000 light years away, convert these measurements into centimeters.
* Measurements reveal that S2 completes one orbit in 15.2 years. Use Kepler’s third law to find the mass of the black hole. How many solar masses does this correspond to?

What is the evidence that we have really discovered a black hole at the galactic center? Clearly something is exerting a force of gravity on these stars, and clearly this force is huge. But does the force have to come from a black hole?

Let us explore the possibility that the object lying at the galactic center is not a black hole after all. Maybe it is something else! The only way to evaluate this possibility is to think of what this “something else” might be and examine the consequences.

Let us call our unknown and unseen thing “Object X.” So far, all you know about it is its mass. But there is something else that you can find out. Notice that the orbit of S2 carries it very close to Object X. Clearly, Object X must be smaller than S2’s distance of closest approach to it: otherwise, S2 would have already collided with Object X and been sucked into it.

* From your above measurement of S2’s orbit, calculate this maximum possible radius of Object X.

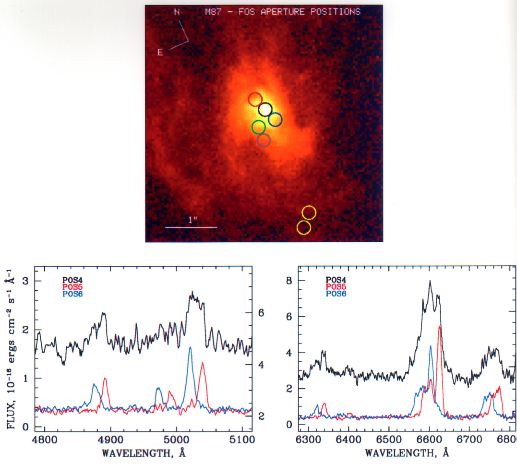
There are a lot of things we might propose for our purported Object X. Let us start by taking the guess that Object X might be a great number of perfectly ordinary stars packed into a small region of space.

* Is there enough room in Object X for the required number of stars? (A typical star such as the Sun has a radius of 700,000 kilometers.)
* How much light would be emitted by such a group of stars – more light than is actually observed coming from the galactic center? You will have to search the Web for observations of the galactic center: see what you can find. Do such observations rule out this possibility?
* These stars cannot be simply sitting there: they must be orbiting one another. You can get a rough idea of their orbital velocity by imagining your group of stars to be a single “thing” in whose gravitational field a single star is orbiting. What would be the velocity of such an orbit?
* Would these orbiting stars, moving at such a velocity, be expected to collide with one another? How violent would be the collisions – violent enough to disrupt the stars?
* The remainder of this problem is very free-form. Continue your analysis of our suggested possibility for Object X. Alternatively, think of some other things that it might be. For each alternative, what constraints are supplied by your measurements? There is no set form for this part of the problem -- be creative!

A Black Hole at the Center of Messier 87

As perhaps you know, a giant black hole has been found at the center of the galaxy M87. In this problem, we go through the evidence that initially lead people to this conclusion.

Here is an image of the very central regions of this galaxy. Notice the elongated glowing structure lying there:



4600 5000 5400 5800

Wavelength (Angstroms)

Concentrate on these lines

2 arc sec

Distance to this galaxy: 53.5 million

light years.

Spectra were taken at various points within this structure. Concentrate your attention on the spectral lines coming from the two indicated regions. As you can see, the two spectral lines are at somewhat different wavelengths, indicating that the two regions have different velocities relative to us. We interpret this to mean that the structure is rotating.

More properly, each of these two indicated regions is orbiting about some unseen mass lying at the heart of the rotating structure. By using Newton’s formula for circular orbit velocity, we can use the orbital velocity to find the mass of this unseen object. We will find that the mass is so large that it can only be a black hole.

* Find the distance each of these regions lies from the center of the structure. Give your answer in cm and light years. (M87 is 53 million light years away.)
* Find the difference in wavelength between the spectral lines coming from these two regions.
* Explain clearly why this difference equals, not but 2  where  is the quantity appearing in the Doppler effect formula v/c
* Find the velocity of rotation – or rather, orbit – of the structure. Give your answer in cm / sec, but then convert it to miles per hour to get an intuitive feel for it.
* Find the mass of the unseen object the regions are orbiting about. Give your answer in gm, but then convert it to solar masses.
* If this unseen object were in fact this many stars, what luminosity would you predict to be coming from the central regions of this galaxy?

(The answer you will get from this last calculation is very much greater than the actual luminosity observed from these regions. This leads us to the conclusion that the unseen object at the center of the rotating structure is a black hole.)

Stars Falling into the Black Hole in Messier 87

A black hole has been discovered at the center of the galaxy M87. In this problem, we study how this black hole can account for the great amount of energy emitted by this galaxy.

Observations reveal that M87 emits about 10 43 ergs of X-ray energy each second. We will suppose that this emitted energy arises from stars that are being sucked into the black hole: as they fall, they will be torn apart and emit huge amounts of energy. We know that no energy can escape a black hole – but energy can certainly escape an object as it falls into the hole prior to reaching the hole’s edge. So the theory we will explore here is that every so often a star falls into this black hole: the energy each star emits is what powers the emission from Messier 87.

Our goal in this problem is to calculate how often the hole must swallow a star if this theory is correct.

Calculations suggest that if a star of mass Mstar falls into a black hole, about

Estar = 0.1 Mstar C2 will be emitted. In what follows, suppose the stars are just like our sun.

* How many ergs will be emitted if a star the mass of the sun falls into the black hole in Messier 87?

All this energy is emitted in a single burst – a spray of electromagnetic radiation and rapidly-moving particles which are injected into the magnetic field filling the entire galaxy. These particles then steadily emit energy until all their energy is used up.

* Given that these particles are emitting X-ray energy at the rate 10 43 ergs/second, how much time elapses before they have radiated away all that energy Estar? The answer that you get to this question is the time interval between successive “swallowings” of a star: calculate this time in seconds, but then convert it to years.

(For your information: more detailed models, including all forms of emitted energy, suggest that a sun’s worth of mass is swallowed by the hole every 10 years.)

RELATIVITY AND COSMOLOGY

A Small Black Hole

Normally when discussing black holes we think of objects whose mass is that of a star or far greater -- a hole formed by the collapse of a star that has run out of nuclear fuel, for example, or the sort of hole which is found in the core of a galaxy. But holes of smaller mass are also theoretically possible. In this problem, let us think about one 2 inches in diameter: its Schwarzschild radius is then Rsch = one inch. (Remember that the Schwarzschild radius of an object of mass M is

Rsch = 2 G M / C2)

* What is this hole’s mass?

Now suppose that such a hole drifts into the room you are now sitting in. Suppose it is ten feet away from you.

* Find the force of gravity this hole exerts on you (suppose that your mass is 70 kilograms). Give your answer in pounds.
* Now imagine your twin sister, who lives 2,500 miles away. She has the same mass as you, but she’s further away from the hole. Calculate the force the hole exerts on her.

The results you get from both these calculations will show that both you and your sister will be sucked into the hole. Now imagine the ocean. To be specific, let’s think about just a bit of the ocean: a cube of seawater, one centimeter on a side, located 1,000 miles away.

* Given that water’s density is 1 gram / cubic centimeter, find its mass and the force of gravity our hole exerts on this bit of the ocean.
* To get an intuitive feel for your answer, calculate the weight of that bit of seawater – that is, the force of gravity upon it from the Earth as a whole. Find the ratio between the force on the seawater from the hole to that on the seawater from the Earth.

The result you will get is that the hole exerts a far larger force than the Earth does on that bit of seawater. As a matter of fact, if you were to keep on repeating these calculations for other things, you would keep on getting the same answer: that little hole exerts a truly enormous force upon the Earth. What this means is that the hole will suck into itself the entire Earth.

Once it has done so, its mass will have grown larger: its mass will ultimately become its original mass plus that of the Earth. This means that the hole will have grown larger.

* Find its new Schwarzschild radius.

How Hubble Found His Law

Every textbook describes how Hubble discovered his law of the expansion of the universe. But sometimes the presentations are a little vague. Here we will go through the actual process, in order for you to get a clearer idea of how he did it.

Hubble made his discovery by measuring the distances and recession velocities of a lot of galaxies: when he graphed velocity against distance, he found the result to be a straight line. The equation for that line

Velocity = (Hubble’s constant) (distance)

is Hubble’s law.

Hubble found the distances by using the Cepheid variable technique. Imagine that you have observed three galaxies: A, B and C. In each of these galaxies, you have found a Cepheid variable star. You have measured the period of variation of each Cepheid, and the average apparent brightness – the flux -- of each of them. Here is what you found:

|  |  |  |
| --- | --- | --- |
| Galaxy | Cepheid Period (days) | Average flux (ergs/sec-cm2) |
| A | 3.2 | 1.60 X 10 -15 |
| B | 8 | 1.19 X 10 -16 |
| C | 6 | 3.71 X 10 -17 |

As you know, by observing Cepheid variable stars within an object and measuring their period of variation and average flux, you can determine the distance to that object by using the period / luminosity relation:

* Use this period / luminosity relation to find the luminosity of each Cepheid variable star.
* Use the measured average flux of each star to find the distance to each of the galaxies in the above table.

Hubble measured the velocities by using the Doppler effect. Now you observe the wavelengths of light emitted by these galaxies. We will say that you know this light should have a wavelength of 5000 Angstroms. But you find that it doesn’t: the wavelengths you observe turn out to be slightly longer than that. Here is what you find:

|  |  |
| --- | --- |
| Galaxy | Observed Wavelength (Angstroms) |
| A | 5002.5 |
| B | 5012.2 |
| C | 5019.5 |

• Find the velocities of these galaxies.

* Graph the velocity versus distance for these three galaxies. Do these galaxies obey Hubble’s law that velocity is proportional to distance?

• Find the value of the Hubble constant from this data. Express it in the “mixed” units of kilometers / second per megaparsec.

Hubble’s Law is Universal:

A Different Law Than Hubble’s Would Not Have Been Universal

Hubble’s discovery of the expansion of the universe might make you think that we are the center of the cosmos. After all, galaxies are flying directly away from us, which seems to say that we lie at the very center of the expansion.

In fact this is not so: we are not the center of the cosmos. In this problem we will see how this can be.

We will do so by analyzing the motions of galaxies away from us – and then we will study how they move away from some other galaxy. We will imagine an alien astronomer, living in that distant galaxy, and we will ask what the alien would observe. What we will discover is that the alien would observe the same thing we do. We summarize this by saying that Hubble’s law is universal: it applies to every observer.

The consequence is that both we and the alien are tempted to conclude that each of us lies at the center of the cosmos. Of course this cannot be true. In reality, there is no center of the cosmos.

Here is the demonstration:

Suppose we observe 6 galaxies: Galaxy A, B, C, D, E and F. We measure their distances D (in megaparsecs) and velocities from us V (in kilometers / second). Here’s what we find

**HUBBLE’S COSMIC TABLE**

|  |  |  |
| --- | --- | --- |
| Galaxy | D (megaparsecs) | V  (kilometers / sec) |
| A | 3 | 210 |
| B | 2 | 140 |
| C | 1 | 70 |
| D | 1 | 70 |
| E | 2 | 140 |
| F | 3 | 210 |

Now we do a little math: we divide V by D and we get 70 in every case. So we conclude

V = H D

where H = 70. This, of course, is Hubble’s law.

Here’s a diagram illustrating what we have found:

210 140 70 70 140 210 V (km/sec)

A B C us D E F

3 2 1 1 2 3 D (mpc)

Now we want to ask a new question: what would an alien astronomer living on another galaxy find? To be specific, let’s suppose that our alien’s name is Elbbuh, and it (they don’t have sexes) lives in galaxy D. We need to find

1. the *distances Elbbuh measures.* These are the distances of each galaxy from D. We will fill some entries in the table: you fill in the rest

|  |  |
| --- | --- |
| Galaxy | Distance from galaxy D (mpc) |
| A |  |
| B |  |
| C | 2 |
| Us | 1 |
| E | 1 |
| F |  |

(b) the *velocities* *Elbbuh measures.* These are the velocities of all the galaxies relative to D.

These are a little harder to find. Let’s start with “us” -- this is pretty easy. Since D is going 70 km/sec away from us, we are going 70 km/sec away from it.

Now let’s study galaxy C. In our above diagram, it is going 70 km/sec to the left, and D is going 70 km/sec to the right. So the velocity of C relative to D is 70+70=140 km/sec.

Similarly, in the above diagram, E is going 140 km/sec to the right, and D is going 70 km/sec to the right. D is “trying to keep up with E but not going fast enough to do so.” So the velocity of E relative to D is 140-70=70 km/sec.

Here are our results so far:

**ELBBUH’S COSMIC TABLE**

|  |  |  |
| --- | --- | --- |
| Galaxy | Distance from galaxy D (mpc) | Velocity relative to galaxy D (km/sec) |
| A | 4 | ? |
| B | 3 | ? |
| C | 2 | 140 |
| us | 1 | 70 |
| E | 1 | 70 |
| F | 2 | ? |

You fill in the rest of the table

Now here’s the final step. Let’s divide the velocity by the distance to find “Elbbuh’s law.” If we do so, we get 70 in every case. This proves that

Velocity relative to D = (70)(Distancerelative to D)

Notice that Elbbuh’s law is the same as Hubble’s law. This proves that Hubble’s law is universal.

This is all very well. But it turns out that it is true only if the universe is expanding according to Hubble’s law. Had the universe been expanding according to some other law, that other law would not have been universal, and what the alien observes would not have been the same as what we observe.

The way to see this is to use exactly the same logic, but to apply it to a different law of expansion.

* Carry out this process, supposing that Hubble had found that V = HD3. Find the distances DElbbuh and velocities VElbbuh that Elbbuh would have observed, and show that it is not true that VElbbuh = H DElbbuh3

Of course you have shown this to be true only for one possible law. It turns out that you can show mathematically that V = HD is the only law which has this “universal” nature! The concusion is that, among all the possible ways the cosmos might expand, only one is universally true – and that is the way the cosmos actually does expand.

There are two attitudes we might take about the significance of this:

1. This is a very deep fact about the universe
2. It’s a cute discovery, but it is not teling us anything that is very significant.

Which is your attitude – and why?

Geometry and the Thermal Expansion of Solids

You may be familiar with Einstein’s argument that acceleration induces a change in geometry. It is one of the foundations of his theory of general relativity.

Einstein’s argument rests on the Lorentz contraction: the fact that a meter stick changes its length when moving. He showed that when the motion is an acceleration (centripetal acceleration), the consequence is to distort geometry. In order to more clearly understand his argument, we will explore in this problem the geometrical consequences of another way in which a meter stick can change its length.

It is well known that most solids expand upon heating, and contract upon cooling. Let us imagine a meter stick whose length is

* one meter at a temperature of 0 degrees F
* ½ meter at a temperature of minus 5 degrees F

Now imagine a race of ants which live on a flat surface which has different temperatures in different places. Suppose the “weather map” of the ant’s world is as follows:

Minus 5 degrees outside

0 degrees

inside

R

R

You will show that the geometry of this world is non-Euclidian.

Begin by thinking about a circle whose radius is less than R – the dotted circle:

The ants measure its radius and circumference in the normal manner– by laying out meter sticks, and counting how many sticks it takes to cover the radius and circumference. Dividing their measured circumference by the radius, they find the answer to be 2π -- all perfectly familiar.

But now let’s think about a circle whose radius is bigger than R. Now, when the ants measure the radius and circumference of their dotted circle, their meter stick is going to contract and expand. However, the point is that the ants don’t know this: they regard their meter stick as the definition of the meter, no matter where it is and what temperature it is. As a result, they are going to get non-Euclidian results.

* Suppose first that the dotted circle is just infinitesimally bigger than R. Prove that the ratio between the circumference and the diameter is not π, but 2π:

R

Finally, suppose that the dotted circle is a lot bigger, so that it looks like this:

R

D

* Prove that the ratio between the measured circumference and diameter is not π but



LIFE IN THE UNIVERSE

Probability: Unusual People

You may have heard of the “Drake Equation,” a formula which gives the number of extraterrestrial civilizations in the Galaxy with which we might exchange messages. The logic behind the Drake Equation proceeds by listing the conditions required for such a civilization to exist (a star must have a planet, that planet must be warm enough to support life, life must have arisen, and so forth). It then assigns probabilities to each of these conditions.

Here is a problem using the same logic.

Suppose that only 1 out of every 20 people had red hair, that half of these red-heads were female, and 1 out of 3 of these were left-handed.

* What is the probability that a randomly chosen person is

a left-handed female red-head?

* Look up the population of the USA, and its surface area.
* How many left-handed female red-heads are there in the USA?
* How many people do you have to look at before you find a left-handed female red-head? Let’s call this number Nsearch

Now we want to find out how far away is the nearest such person (if they are distributed uniformly across the nation). The way to think of this is to imagine a map, and on it a dot representing your location. Now draw a circle on that map, with you at its center. This circle encloses a certain number of people: if you begin with a small circle, and then make it bigger and bigger, it will enclose a progressively larger and larger number of people.

* What is the number of people per unit area in the USA?
* How big does your circle have to be before it encloses Nsearch people?

Your answer to the last question tells you how far away is the nearest left-handed female redhead.

The “Timing” Probability in the Drake Equation

You may have heard of the “Drake Equation,” a formula which gives the number of extraterrestrial civilizations in the Galaxy with which we might exchange messages. An element in the Drake equation is the so-called “timing probability:” the probability that the communicating phase of an alien civilization coincides with now.

We imagine a star, about which orbits a planet. We imagine that life arises on that planet, and ultimately a technologically advanced civilization. But this might have happened billions of years ago, and the civilization by now might have been wiped out. Alternatively, perhaps it has not happened yet. If we wish to exchange messages with an extraterrestrial civilization, we need its ability to communicate across interstellar space to have arisen right now, just as we have begun searching for its signals. What are the chances that this is the case?

Here’s a diagram:

T star

Tcommunicating

Star forms the ability to Now the ability to Star ‘dies’

communicate communicate

begins ends

Although I have indicated a particular time for “now” in this diagram, in reality “now” can fall at any moment within the lifetime of the star. The “timing probability” in the Drake Equation gives the probability that “now” falls at some point within the communicating phase of the alien civilization: this probability is

Ptiming = Tcommunicating / T star

Here’s an Earth-bound situation that uses exactly the same logic:

Mary regularly goes to bed at midnight and gets up at 8 in the morning. But she is an insomniac. Every night she wakes up for half an hour. But the times at which she awakens are random: sometimes she awakens after only a few minutes of sleep, at other times just before she finally climbs out of bed, and at other times she is awake in the middle of the night.

* What is the probability that she is awake at 3:15 AM?
* Suppose you yourself awaken at random times during the night – but only for an instant, after which you fall right back to sleep again. What is the probability that, the moment you awaken, she is awake?
* How many nights pass before there is a pretty good chance that (a) Mary is awake at 3:15 AM, and (b) you find her awake when you awaken?
* Would your answers to the above questions be different if, when you awakened, you did not fall to sleep immediately, but rather stayed awake for a while?
* So far we haven’t been thinking about the search for extraterrestrial civilizations (SETI). But now, start doing so. Please translate the above questions into a form relevant to SETI.

Beamed Radiation: “Alpha Centauri Calling”

Let us imagine that an intelligent civilization exists on a planet orbiting Alpha Centauri, and that they want to send us a message by optical laser. They plan to blink their laser on and off in a kind of Morse Code, and they need it to be so bright that we on Earth will be able distinguish it from the glare of Alpha Centauri itself. How bright must their laser be?

In order to figure this out, we need a formula which tells us the flux we receive, not from a source which emits light in all directions, but one which emits into a narrow beam. This means that we will have to re-think the Inverse Square Law.

The Inverse Square Law relates luminosity and flux. Recall the definitions of these terms: luminosity is the amount of energy emitted by a light source per second, and flux is the amount of energy received per second by a detector of surface area 1.

• Explain why these definitions are still perfectly good in this new situation we are discussing now.

Suppose we are in a darkened room, and that a laser is a distance D away from a wall. Suppose that we are shining the laser directly at the wall, and that the laser emits its light into a beam of opening angle  as shown:

wall



Distance D

• All the light emitted by the laser ends up illuminating a circle of a certain radius on the wall. Find a formula for this radius in terms of D and .

• What is the surface area of this illuminated circle?

• Find the formula relating flux to luminosity for this situation in terms of D and .

Now let’s see why narrow beams are good. Let’s consider a100-Watt light bulb,. Suppose we have two options: (1) just turn on the light bulb, or (2) turn on the light bulb, and bounce its light off a reflector so that it forms a narrow beam pointing right at you. To make things specific, let’s suppose that the beam has an angle  of 10 degrees.

* Suppose you are 100 meters away from the light bulb. Calculate the flux you receive from it in both of these two cases.

As you can see, beamed radiation is far more intense than unbeamed radiation.

Finally we are ready to tackle our real question: if the alien’s laser sends light off in a beam of opening angle 1 second of arc --

* What does the luminosity of this laser have to be in order that the flux we receive from it be greater than the flux we receive from Alpha Centauri? [Alpha Centauri’s luminosity equals that of our sun.]

Recently I was looking at the website for the National Ignition Facility. This is a project attempting to produce controlled nuclear fusion by pointing ultra-bright lasers at a target. You should check out this website – it’s fascinating.

* The website mentions that their lasers produce a luminosity of 500 trillion Watts. Do the aliens on Alpha Centauri need a laser even more powerful than this?

A Message From Space

On the night of January 1-2, 2039 a flickering light was detected from the star Vega. The flickering consisted of pulses of light of two durations: a short pulse lasting 2.3 seconds, or a longer one lasting 11.6 seconds. Using an “S” to denote a short pulse, and an “L” to denote a long one, here is the pattern:

LSLLSLLLSLLLLSLLLLLSLLLLLLSLLLLLLLSLLLLLLLLSLLLLLLLLLSSSSSSSLLLLLLLLLSSLLLLLLLLSSLLLLLLLSSLLLLLLSSLLLLLSSLLLLSSLLLSSLLSSLSSSSSSSLSLLSSSSSSSLSLLLLLSSSSSSSLLSLLLSSSSSSSLLSLLLLLSSSSSSSLLLSLLLLLSSSSSSSLLLLSLLLLLSSSSSSSLLLSSLSSSSSSSLLLLLSSLSSSSSSSLLLSSLLSSSSSSSLLLLLSSLLSSSSSSSLLLLLSSLLLSSSSSSSLLLLLSSLLLLSSSSSSSLSSSLSSSSSSSLLSSSLLSSSSSSSLLLSSSLLLSSSSSSSLLLLSSSLLLLSSSSSSSLLLLLSSSLLLLLSSSSSSSLSSSSLSSSLLSSSSSSSLSSSSLSSSSLSSSLLLSSSSSSSLSSSSLSSSSLSSSSLSSSLLLLSSSSSSSLSSSSLSSSSLSSSSLSSSSLSSSLLLLLSSSSSSSLSSSSLLSSSLLLSSSSSSSLLSSSSLLSSSLLLLSSSSSSSLLSSSSLLLLLLSSSLLLLLLLLSSSSSSSLLLLLSSSSSLSSSLLLLSSSSSSSLLLLLSSSSSLLSSSLLLSSSSSSSLLLLLSSSSSLLLSSSLLSSSSSSSLLLLLSSSSSLLLLSSSLSSSSSSSLLLSSSSLLLLLSSSLLLLLSSSSLLLSSSSSSSLLLSSSSLLLLSSSLLLLSSSSLLLSSSSSSSLSSSSSSLLSSSSSSLLLSSSSSSLLLLSSSSSSLLLLLSSSSSSSLLLLLSSSSSSLLLLSSSSSSLLLSSSSSSLLSSSSSSLSSSSSSSLLLSSSSLLLLLSSSSSSLLLLSSSSLLLLLSSSSSSSLLLSSSSLLLLLSSSSSSLLSSSSLLLLLSSSSSSS

The pattern was found to repeat over and over again – but it did not persist indefinitely. Rather it continued for 18 hours and then stopped – but then, 18 hours later, it resumed. This 36-hour on-off cycle has continued since its discovery.

More careful analysis of the laser signals revealed that they were received in a spectral line that ought to have a wavelength of 5,000 Angstroms, but which turned out to have a wavelength that smoothly shifted from 5,000.34 Angstroms to 4,999.66 Angstroms and back again over a period of 13 years.

Scientists studying the transmission have drawn a number of inferences from it. Some of these inferences are iron-clad, but others are fairly tentative. Here they are:

* By considering the nuclear-burning lifetime of Vega, they have concluded that the progress of evolution on the planet orbiting Vega has proceeded more rapidly than it did on Earth.
* They have found that this signal was sent in 2012
* They have found that this planet orbits Vega at 20 kilometers per second.
* They have found that the length of this planet’s year – the time it takes to orbit Vega – is 13 of our years.
* They have found that there are 3,165 of this planet’s days in its year.
* They have concluded that the creatures sending the transmission have ten fingers (or ten toes, or ten legs – ten “somethings” at any rate)

Your task is to repeat the process that these scientists went through, and to derive all these results yourselves.

All this, however, is only the beginning. The exciting thing about this transmission, the scientists concluded, is that it must be from an intelligent civilization. Their evidence for this conclusion is that the transmission contains a message.

* Decipher the message, and figure out what the aliens are trying to tell us.

*[NOTE: Vega shines by converting Hydrogen to Helium. Its mass is 2 times the mass of our Sun, its luminosity is 37 times the luminosity of our Sun, and its parallax is 0.125 seconds of arc.]*

Finally, here is a puzzle that you might find amusing: Something in my formulation of this problem is astronomically impossible. Indeed, I have intentionally inserted an error into my description of the situation. Can you find this error?

*NOTE You might find it difficult at first to decipher the message. At the end of this site there is a hint which might help.*

Three Problems Concerning Supercivilizations

It may be that our technological civilization will survive a very long time – billions of years, let us say. If we look back on the progress of technology over the past few decades, we realize how hard it is to predict what might be possible if we have such immense gulfs of time before us. If you doubt this, ask your parents what technological wonders they possessed when they were in college.

Nevertheless, let us try to imagine some very ambitious engineering projects. In the three problems that follow, we will let our imagination run wild – but we will be very careful to stick to the laws of nature. Our philosophy will be that if something does not actually violate a law of nature, then it is possible.

As you work through these problems, you may feel that they are all crazy. I myself do not agree: if we have billions of years to do things, the most amazing things are possible. And here’s another point: this may have been done already – not by us, but by some alien civilization. Perhaps we should search for extraterrestrial civilizations in a new way: by looking for the signs of such gigantic engineering projects.

Ring World

The first project we envision will be one designed to give humanity more living room, and more solar energy. We will do this by taking apart the entire planet Earth, and remaking it.

First we will assemble the material of the Earth into a long thin strip of length d, width h and thickness t:

t

h

d

And then we will bend it into a ring entirely surrounding the sun:

length d all the way around

h

Radius of ring R = 1 Astronomical Unit

(I’m not good enough at drawing to indicate the thickness t, but you get the point.)

We will live on the inside surface of the ring[[1]](#footnote-1). If we make the radius of the ring equal to1 Astronomical Unit, the temperature on this inside surface will pretty much be the same as that of our Earth, which is what we want.

* Why won’t the temperature be exactly equal to that of the Earth? Do you expect it to be hotter or colder than this?

We will have built an artificial world. Because the ring has a far greater surface area than the Earth, it will give us more room to live, so that the ring will support a far greater population than our poor old Earth does right now. Furthermore, remember that half of our Earth is always in darkness, and lots of the Earth is oriented at a slant angle to the incoming sunlight, making it an inefficient collector of solar energy. But notice that in our new artificial world, the entire inside surface of the ring faces the sun, making the ring a perfect collector of solar power. So humanity will have a far greater source of solar energy than it does now.

Our goal in this problem is to explore various aspects of this project.

First, let us ask how thick the ring can be. Let us suppose that, once we have built it, we will have used up all the matter in the entire Earth. Let us also suppose that the ring will be made of the same sort of matter – namely, rock – as the Earth. In that case, the volume of matter making up the ring must equal the volume of the Earth.

* Suppose we make the ring’s width h equal to 100 miles. Given that the volume of such a ring is just that of the strip we began with (before bending it into a ring), find the ring’s thickness ‘t.’ Express your answer in miles to get a feel for it.
* Find the surface area of the inside surface of the ring.
* Find the ratio between this area and the surface area of our Earth.

Look up on the internet the present population of the Earth. Suppose the number of people per acre in this new world is the same as now: then what you just found is also the ratio between the new world’s population and the present population.

* How many people can this world support?
* Look up on the web the total amount of solar power falling on the Earth. How much solar power would fall on this new world?

Now let us think about “artificial gravity,” which will come from the rotation of the ring. If we set the ring rotating with velocity V, then everything on its inside surface will be moving with velocity V in a circle of radius R about the Sun. So everything will experience a centripetal acceleration

Acent = V2 / R

If we set this equal to the acceleration of gravity we experience right now, which is 980 cm / sec2, then everyone living on the inside surface of the ring will experience an “artificial gravity” which feels like the real gravity we experience.

* Find the velocity V which we must give the ring in order for this to be so. Express it in millions of miles per hour to get a feel for it.

Now let us ask how much energy will be required to bring the ring up to this speed.

* Remembering the formula for kinetic energy of a body of mass M moving with velocity V [K.E. = (1/2) M V2] calculate the energy required.

Where are we going to get all that energy? Let us propose that we will get it from solar power. Before taking apart the Earth, we will accumulate the required energy by coating the whole Earth with solar power collectors, and trapping every bit of solar energy that falls upon it.

* At this rate, how long would it take to accumulate the required amount of energy? Express your answer in billions of years to get a feel for it.

Your answer shows that this is going to take a long time. Let us try to imagine a faster way. It involves a two-step process. First we will completely surround the Sun with a gigantic spherical solar collector, trapping all the energy the Sun emits. Then we will use this accumulated energy to set our artificial world spinning.

* At this far greater rate of trapping solar power, how long would it take to accumulate the required amount of energy? Express your answer in years to get a feel for it.

Interstellar Beacons

The search for intelligent life in the universe is bedeviled by the fact that, in order to find such a civilization, we must survey a great many stars. For our second problem we try an alternate approach: instead of searching for aliens, we will do something which draws the aliens’ attention to us. An analogy to this might be the strategy of searching for people in the woods by giving a loud shout, in hopes that the shout will draw their attention.

This problem analyzes one such strategy: we will erect a “beacon on the Sun.” The “beacon” will be a sudden change in the Sun’s spectrum. If any extraterrestrial creatures are monitoring the spectra of stars, they might notice the sudden change and come to investigate what has caused it.

The Sun is composed primarily of Hydrogen: within it, heavier elements are very rare. Any given heavy element exists in concentrations of roughly one part in a thousandwithin the Sun. The strategy of an interstellar beacon is to push a big chunk of rock into the Sun. The rock will immediately vaporize, and will be spread about within the Sun. Being composed of elements other than Hydrogen, it will suddenly change the Sun’s spectrum.

Let us agree that, in order to significantly increase the quantity of heavy elements in the Sun, we need to double this quantity. The question is: how big a chunk of rock do we have to push into the sun in order to accomplish this? Concentrate first on the Sun as a whole. A crude calculation shows that we will need to shove into the Sun something with a mass equal to 10 -3 of the Sun’s mass.

* What mass does this work out to?
* If the chunk is made of rock, its density will be perhaps two grams per cubic centimeter. If we suppose the object is spherical, what is its radius?

Your answer shows that this is going to be a difficult project. But here is an easier way. Initially the heavy elements from the chunk of rock are going to be confined only to the outer layers of the Sun. So we will only need to push into the Sun a chunk with a mass equal to 10 -3 of the mass, not of the entire Sun, but of these outer layers. This region has a thickness of about 1,000 kilometers: it is a thin spherical shell of thickness 1,000 kilometers and radius equal to that of the Sun.

* What is the volume of this region of the Sun?
* The density of the Sun’s outer layers has been measured to be about 10 -7 grams per cubic centimeter. Find from this the total mass of the Sun’s outer layers, and the mass of the required chunk.
* Now what is the radius of the chunk?

Convert your answer (which will be in centimeters) into some more comprehensible units. What sort of object has this radius?

Push a Planet into the Sun

Finally we take a different tack on the previous problem, and consider the task of pushing a planet into the Sun. To be specific, let the planet be Jupiter. Your task in this problem is to see how hard this will be.

Of course we don’t really need to push Jupiter into the sun. What we really need to do is stop Jupiter in its orbit: once we do so it will simply fall into the sun. So your real task is to analyze how hard it will be to stop Jupiter’s orbital motion.

We will do this by mounting a giant rocket engine on Jupiter, and set it firing:

The rocket engine fires this way

Jupiter orbits this way

The force from the engine pushes Jupiter this way

The biggest rocket engine we are currently capable of building produces a force of about 4 million pounds. Let us suppose that our “planetary rocket engine’ is one million times more powerful than this. We travel out to Jupiter, set it up, and turn it on. The engine keeps on firing and firing, steadily slowing Jupiter’s motion.

* Use Newton’s law F = MA to find the acceleration this produces.
* Jupiter is 5.2 AU from the Sun and its orbital period is 11.87 years: from these, calculate its orbital velocity.
* Since acceleration is the rate of change of velocity, the time “T” required for an acceleration “A” to bring Jupiter from this velocity to zero is T = V / A. Calculate the time it will take this rocket engine to bring Jupiter to rest.

Your answer will show that this is not going to be good enough: the time you will get turns out to be longer than the lifetime of the sun! This means that the sun will have become a red giant, wiping us all out, long before we manage to push Jupiter into it. So now you will have to go back and re-do your calculation:

* How big a rocket engine do we need to set up on Jupiter if it is capable of bringing Jupiter to a stop in one billion years? Give your final answer in pounds of force it exerts. Find also the ratio between this force and the biggest engine we are capable of building at present.

Finale

The Hint to “A Message From Space”

You may have read of other “interstellar messages” which encode some sort of picture, and you may be tempted to think of this message in the same way. In fact you will get howhere working in this way: that’s not what the message is about at all!

So what is the message? Here’s a hint: take a look at its first line. It consists of first one Long, then two Longs, and so on. So clearly the message begins by counting. This is a clue that the message involves mathematics, specifically the mathematics of the integers.

1. The science fiction author Larry Niven explored such a world in his novel “Ringworld.” [↑](#footnote-ref-1)